Eddy Viscosity Models for Wakes and Separated Flows

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Introduction

WITH the increasing popularity of differential equation models of turbulence one tends to question the usefulness of eddy viscosity models for computing turbulent flows. It is clear that eddy viscosity models cannot give details of the turbulence structure, but can predict overall features of a flow. In this Note we discuss some of the results obtained during our computational investigations of complex trailing-edge flow problems utilizing eddy viscosity models and attempt to recognize some of their merits and drawbacks.

Here we consider the flow past the trailing edge of a flat plate and the flow past a backward-facing step. The first flow is a standard flow involving a wake. The second one has become a benchmark problem for flow separation and reattachment.

Governing Equations and Computational Details

The flows were computed by solving the two-dimensional unsteady compressible Navier-Stokes equations by a pseudotransient, time-split finite element formulation. The Mach number considered was 0.4. Srinivas and Fletcher[1] give a detailed account of the governing equations, assumptions, and boundary conditions. The governing equations were solved by a time-split finite element method[2] with linear, rectangular elements. An ADI formulation was used to solve the resulting equations.

Eddy Viscosity Model

The eddy viscosity model employed closely follows Ref. 3 and is not given here. The model applied in the separated region is slightly different from that in Ref. 3 and is as follows.

The eddy viscosity value outside of the separation bubble is calculated by the formulation for the wake.[3] Within the bubble the value is interpolated between the values at the edge of the bubble and the wall and is damped by the Van Driest damping factor D. Thus

$$\epsilon_i = k_2 U_\delta \delta^* [(y/y_{ds})D]^N \tag{1}$$

with $N=1$, whereas in Deiwert's formulation $N=2$. Here, ϵ is the eddy viscosity, $k_2 = 0.0168$ and

$$\delta^* = \int_{y_{ds}}^{\delta} \left(1 - \frac{u}{U_\delta}\right) \mathrm{d}y$$

where y_{ds} is the dividing streamline.

Relaxation

The eddy viscosity values calculated for the regions downstream of the plate and the backward-facing step were relaxed by using the following formula:

$$\epsilon_{i,j} = \alpha \epsilon'_{k,\ell} + (1-\alpha)\bar{\epsilon}_{i,j} \tag{2}$$

where $\bar{\epsilon}_{i,j}$ is the value calculated from the equations[3] and $\epsilon'_{k,\ell}$ is the value at an upstream location k, ℓ. This location, k, ℓ, was obtained by tracing back along the velocity vector at position i, j and the value of $\epsilon'_{k,\ell}$ was obtained by linearly interpolating among the four corners of the element containing location k, ℓ.

Results and Discussion

Flow Past the Trailing Edge of a Flat Plate

The mean u-velocity profiles for various locations downstream of the trailing edge are given in Fig. 1 and these are compared with the profiles obtained by Viswanath et al.[4] from experiments. We find close agreement between our results and those of Viswanath et al. except close to the trailing edge. The disparity is greater near the wake center line than in the outer regions. The velocity defects at the location $x/\theta_0 = 0.8$, in the present study and those of Viswanath et al. are 0.81 and 0.7, respectively.

The disagreement can be attributed to one or more of the differences in the flow situations in the two studies. The Reynolds numbers are different—10^6 in the present study and

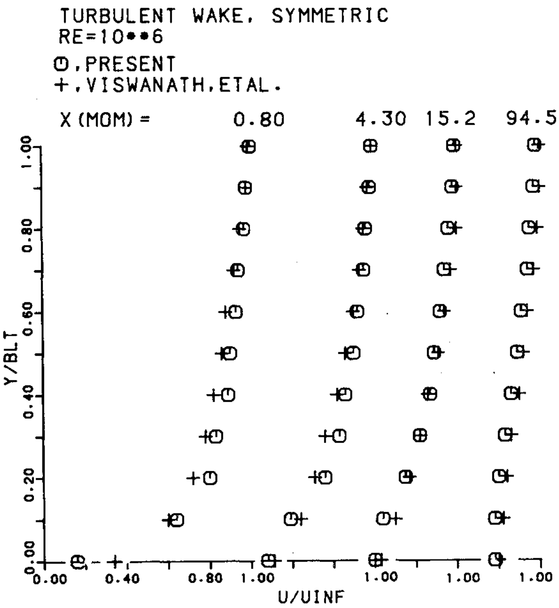

Fig. 1 Comparison of the mean u-velocity profiles for the flow past a flat plate with those of Viswanath et al.[4]

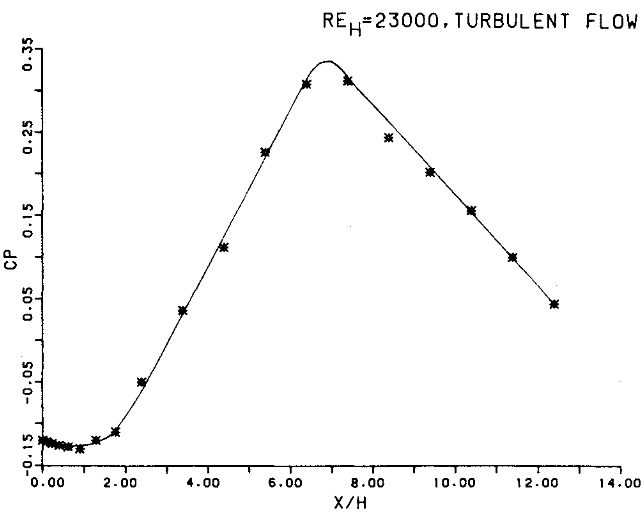

Fig. 2 Computed wall pressure distribution for the flow past a backward-facing step, CP = pressure coefficient.

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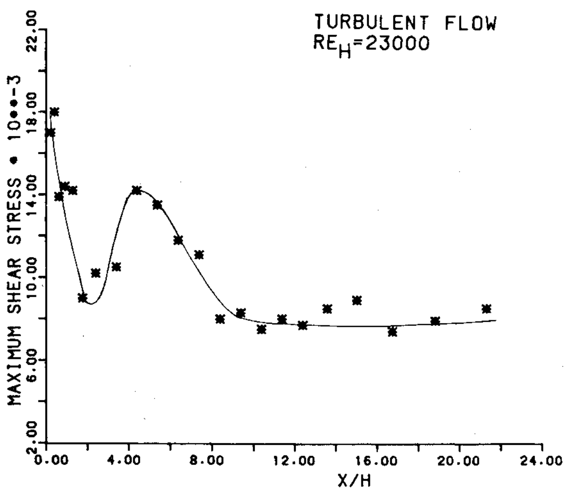

Fig. 3 Comptued distribution of maximum shear stress.

22.6×10^6 in that of Viswanath et al. Second, we consider a flat plate at zero thickness whereas Viswanath et al. consider a 6.25 deg wedge. The effect of the flat plate or the wedge is more pronounced near the trailing edge and attenuates downstream of the trailing edge.

It is also likely that the streamwise gradient of the normal stress, which is ignored in the eddy viscosity formulation, is contributing to lack of agreement. The streamwise gradient of the normal stress rapidly diminishes downstream of the trailing edge.

Viswanath et al. have also computed the flow past a flat plate utilizing a two-equation model of turbulence. Their computed results show good agreement with the experimental results, thus indicating that our results also are in accordance with the computations of the two-equation model.

For the far wake (not studied by Viswanath et al.) our computational results closely match the experimental results of Chevray and Kovasznay[5]; for example, the computed velocity defects at $x/\theta_0 = 258$ and 414 are 0.14 and 0.11 vs 0.17 and 0.11 measured by Chevray and Kovasznay.

Flow Past a Backward-Facing Step

This flow was computed for a step-height Reynolds number, $Re_H = 23{,}000$, and for a step height to inlet boundary-layer thickness of 1.0. The computed wall pressure distribution is shown in Fig. 2 and it demonstrates good qualitative agreement with that obtained experimentally by Chandrasuda and Bradshaw.[6] The maximum shear stress profile (Fig. 3) also exhibits good qualitative agreement with those of experiments considered in Ref. 7. The peak value computed near reattachment is 14×10^{-3}, compared to 11×10^{-3} measured in experiments. In contrast, most of the computations reported in Ref. 7, utilizing differential equation models, substantially overpredict the maximum shear stress.

One of the important features of a flow over a backward-facing step is the length of the separation bubble which was found to be 5.5 step heights in close agreement with the previous experimental studies.[8] This may be contrasted with the differential equation models of turbulence which are reported to considerably underpredict the length of the separation bubble.[7]

Reattachment and the subsequent development of the subboundary layer are other important features of the flow. In the present computations the maximum skin friction coefficient after reattachment is 1×10^{-3} vs 3×10^{-3} upstream of the corner. The skin friction distribution and, hence, the velocity distribution close to the wall and downstream of reattachment are underpredicted. This has also been the observation with the two-equation models although they are better than the eddy viscosity models in this regard.[9]

Conclusions

Our study indicates that eddy viscosity models can compute most features of a wake flow and a separated flow satisfactorily. The main drawback is their inability to predict accurately the shear stress close to the wall downstream of reattachment. This drawback seems to be shared with two-equation models, such as the k-ϵ model.

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Nonequilibrium Arc Modeling

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MANY attempts have been made to derive models that describe the thermal and electrical characteristics of arc behavior.[1] Elenbaas and Heller[2] first attempted to solve the conservative equations for an arc column. They considered an arc column in any asymptotic equilibrium flow regime which leads to a decoupling of the energy and momentum equations. Using the energy conservation equation plus Fourier's law of heat conduction they obtained the Elenbaas-Heller equation. Several authors[3] have presented solutions of the Elenbaas-Heller equation both for constant and temperature-dependent transport coefficients., These solutions allow the determination of the local physical parameters of the plasma and their relation to the external energy input and to the dimensional characteristics of the arc. This information is useful in practical arc design. In many technical applications

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